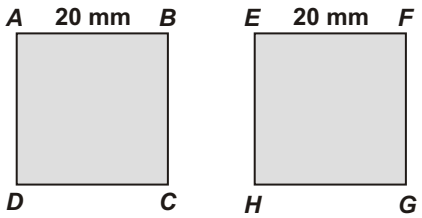
Similar Polygons

Section 21.1
Introducing Similarity

This is part of a conversation overheard on the way to geometry class.

First girl: "Your book bag is just like mine!"
Second girl: "No, it's not!"
First girl: "It is, too. It has the same shape and everything."
Second girl: "But it's a different color."
First girl: "That doesn't matter; they still look alike."
Second girl: "Alike yes, but not exactly the same. Besides, mine is bigger."
First girl: "OK, OK maybe they're not exactly alike — but they are similar."

When it comes to geometry, the idea of objects being exactly alike or just similar is called **congruence** and **similarity**. For objects to be congruent, they must be exactly alike in both size and shape. In similarity, the shape is the same, but the size is different. The first girl was right about one thing: color doesn't matter. Let's look at something a little easier to compare than book bags.

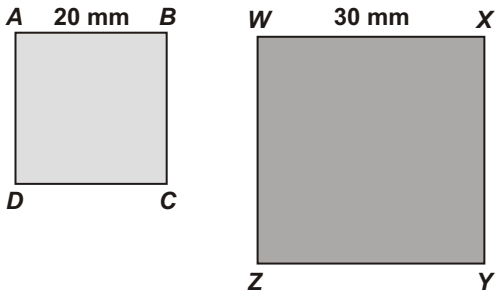


Square *ABCD* is *congruent* to square *EFGH*.

ABCD ≅ ***EFGH***

Consider the shape of a square. In a square, all the sides have the same length, and all the angles have the same measure. If you have two squares with the same side lengths, then everything else is equal as well. You could actually pick up the first square and place it on the second square, and everything would exactly match. If both objects are exactly the same, they're **congruent**. Square *ABCD* is congruent to square *EFGH*. The symbol (≅) means *congruent*.

Now consider the same two squares, but change the side length of one to 30 mm. They still look alike, but they aren't exactly the same. The angles are still the same, but the size has changed. Rather than being equal, the side lengths are now proportional. The squares are now **similar** — alike, but not exactly the same. The symbol for *similar* is (~). (If you don't remember what proportional means, that's okay. We'll explain it again later.)



Square *ABCD* is *similar* to square *WXYZ*.

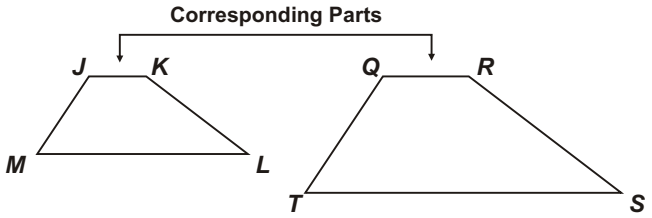
ABCD ~ ***WXYZ***

Corresponding Parts

When two geometric figures are similar, parts on one figure will correspond to parts on the other figure. Corresponding parts can include vertices, sides, and angles. The corresponding sides are not the same length, but they are in the same position on the figure (relative to the rest of the figure).

Polygons (any closed geometric figure with three or more sides) can be named by listing their vertices. When naming a polygon, start at any vertex and list the vertices either clockwise or counter-clockwise. To name a second, similar polygon, the order of the vertices must be the same because the order indicates which point on the first figure corresponds to the similar point on the second figure.

An irregularly shaped polygon may help you to better understand corresponding parts. Look at the two trapezoids on the right. Trapezoid *JKLM* is similar to trapezoid *QRST*. On the diagram, you can see that side *JK* corresponds to side *QR*. Can you list the other corresponding sides? They are *KL* with *RS*, *LM* with *ST*, and *MJ* with *TQ*.

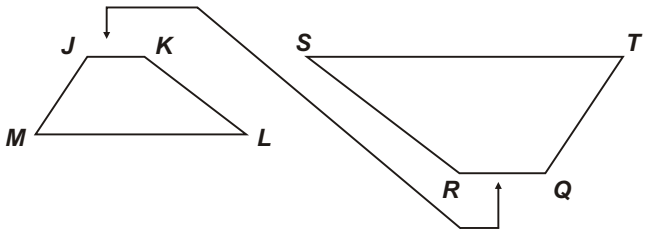


JKLM
↓ ↓ ↓ ↓
QRST

Just by looking at the names, you can also determine which parts correspond to which. The positions of the corresponding points line up in the names. Point *J* corresponds to point *Q*, point *K* to point *R*, point *L* to point *S*, and point *M* to point *T* .

Vertices and sides are not the only corresponding parts. Angles also correspond. The corresponding angles in these similar trapezoids are $\angle J$ and $\angle Q$, $\angle K$ and $\angle R$, $\angle L$ and $\angle S$, $\angle M$ and $\angle T$.

What happens if two similar figures are not oriented the same way? Then looking at the names may help you to identify which parts are similar.



In the diagram to the right, trapezoid *JKLM* is still similar to trapezoid *QRST*, but it's more difficult to "see" the corresponding parts on the diagrams. Vertex *J* still corresponds to vertex *Q*, but since the figures are oriented differently, one is on the top of the figure and the other is on the bottom. Side *JK* still corresponds to side *QR*. Angle *M* still corresponds to angle *T*.

Which side corresponds to *KL*? Look at the names. *KL* corresponds to *RS*.

JKLM ~ QRST

Practice

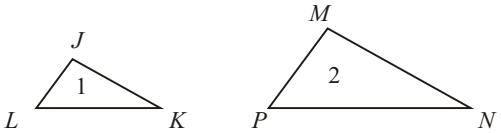
Each pair of figures below is similar. For each side or angle given on figure 1, identify the corresponding side or angle on figure 2. The first one is done for you as an example.

triangle *JKL* ~ triangle *MNP*

MN 1. Side *JK*

_____ 2. Side *KL*

_____ 3. Side *JL*

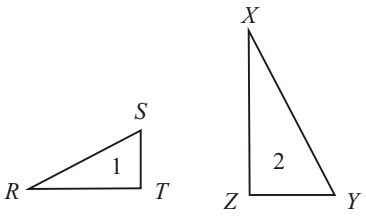


triangle *RST* ~ triangle *XYZ*

_____ 4. Side *ST*

_____ 5. Side *TR*

_____ 6. Angle *R*



trapezoid *ABCD* ~ trapezoid *WXYZ*

_____ 7. Side *BC*

_____ 8. Side *AD*

_____ 9. Side *CD*

_____ 10. Angle *D*