

MAAP-EOC Exam

Algebra I Student Review Guide

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*2016 Mississippi College- and
Career-Readiness Standards for Mathematics*

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Preface

The *MAAP-EOC Exam Algebra I Student Review Guide* is written to help students review the skills needed to pass the Algebra I end-of-course exam in Mississippi. This comprehensive review book is based on the 2016 Mississippi College- and Career-Readiness Standards (MCCRS) for Mathematics as published by the Mississippi Department of Education.

How To Use This Book

Students:

You are required to pass the Algebra I course in order to graduate, and the MAAP-EOC exam for Algebra I factors heavily towards you getting graduation credit for the course. This book covers what you need to know to pass the MAAP-EOC exam in Algebra I.

- Read the instructional material in this review book, do the practice exercises, and take the section review tests at the end of each section.
- After reviewing the material, take the two practice tests (provided as separate booklets). These practice tests are written to look similar to a paper-based Algebra I exam, so they will give you practice answering the types of questions you may see on the exam, whether you take a paper-based exam or the computer version.
- Using the practice test evaluation charts, circle the questions that you answered incorrectly. The evaluation charts will show you where to find the instructional material in the book that corresponds to each question. For each question that you missed on the practice tests, review the corresponding sections in the book.
- If you are using this book as a review either before or after taking the MAAP-EOC exam for Algebra I for the first time, you may want to use one of the practice tests provided as separate booklets to gauge your understanding of Algebra I concepts. Score the practice test. Using the practice test evaluation chart, circle the questions that you answered incorrectly. From the evaluation chart, review the sections in the book that correspond to the questions you missed. Do all the practice exercises and take the section review tests. Then test your skills again by taking the other practice test and repeat the same process.

Teachers:

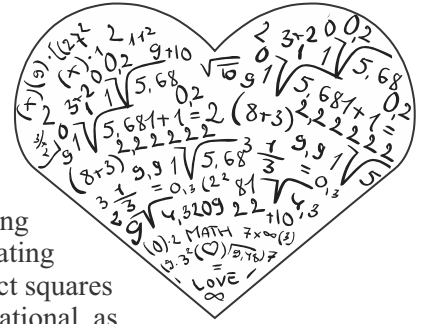
This review book is also intended to save you, the teacher, time in the classroom. It can be used for classroom instruction or for individual student review. Since this student book offers review for ALL of the MCCRS for Algebra I, you have one consolidated resource of materials to help your students prepare for the end-of-course exam.

- When teaching or tutoring individual students, use the strategy outlined above for students.
- For classroom study, use this book to supplement lesson plans and to give additional review for skills required by the MAAP-EOC for Algebra I. Purchase a class set of books for use in the classroom or assign books to students for out-of-classroom work.
- Assign the practice tests (provided separately) as comprehensive review tests. Use the practice test evaluation chart found after each test to identify areas needing further review.
- Please **DO NOT** photocopy materials from this book or the practice test booklets. These materials are intended to be used as a student workbook, and individual pages should not be duplicated by any means without permission from the copyright holder. To purchase additional or specialized copies of sections in this book, please contact the publisher at 1-800-745-4706.

The Real Number System

Section 1.6

Rational vs. Irrational Numbers



As a quick review, *rational numbers* include numbers that can be written as a fraction. When converted to a decimal number, rational numbers have a terminating decimal or a repeating decimal. *Irrational numbers*, however, do not have terminating or repeating decimals. As you've seen, square roots of numbers that are not perfect squares are irrational. The cube root of numbers that are not perfect cubes are likewise irrational, as are any other roots that cannot be further factored from under the radical symbol.

Sums and Products of Rational and Irrational Numbers

When you add or multiply rational numbers, what is the result? Consider the following four rules:

Rules for Adding and Multiplying Real Numbers

- The sum or product of two rational numbers is rational.
- The sum of two irrational numbers is irrational.
- The sum of a rational number and an irrational number is irrational.
- The product of a nonzero rational number and an irrational number is irrational.
- The product of two irrational numbers could be rational or irrational!

These rules are always true. Consider the following examples:

| | |
|--|--|
| <p>rational + rational = rational</p> $\frac{3}{5} + \frac{6}{5} = \frac{9}{5} \text{ or } 1.8$ | <p>rational \times rational = rational</p> $\frac{3}{4} \times \frac{7}{9} = \frac{21}{36} \text{ or } 0.58\overline{3}$ |
| <p>rational + irrational = irrational</p> $\frac{-2}{3} + \sqrt{2} = 0.747546\dots$ | <p>rational \times irrational = irrational</p> $4.2 \times p = 4.2p \text{ or } 13.1946\dots$ |
| <p>irrational + irrational = irrational</p> $2p + 3p = 5p = 15.709\dots$ | |

Adding Square Roots

Terms with square roots that are combined by addition can sometimes be simplified. Remember that the sum of rational numbers is always rational, but if the sum contains an irrational number, the sum will always be irrational.

Example 1: Simplify $\sqrt{4} + \sqrt{9}$. Is this sum rational or irrational?

Both 4 and 9 are perfect squares, so their square roots are both rational numbers. The sum of two rational numbers is rational.

$$\begin{aligned} & \sqrt{4} + \sqrt{9} \\ &= \sqrt{2 \cdot 2} + \sqrt{3 \cdot 3} \\ &= 2 + 3 = 5 \quad \text{rational} \end{aligned}$$

Section 1.6, continued

Rational vs. Irrational Numbers

Example 2: Simplify $\sqrt{9} + \sqrt{8}$. Is the sum rational or irrational?

First simplify each term. The square root of 9 is 3, a rational number. The square root of 8, however, is an irrational number; so the sum is irrational.

$$\begin{aligned}\sqrt{9} + \sqrt{8} \\ &= \sqrt{3 \cdot 3} + \sqrt{2 \cdot 2 \cdot 2} \\ &= 3 + 2\sqrt{2} \quad \text{irrational}\end{aligned}$$

Example 3: Simplify $\sqrt{8} + \sqrt{18}$. Is this sum rational or irrational?

Both of these terms represent irrational numbers, so you know the sum will be irrational. However, they can still be simplified.

Two terms that have the same number under a radical can be combined. The simplified sum is $5\sqrt{2}$.

$$\begin{aligned}\sqrt{8} &\rightarrow \sqrt{2 \cdot 2 \cdot 2} = 2\sqrt{2} \\ \sqrt{18} &\rightarrow \sqrt{3 \cdot 3 \cdot 2} = 3\sqrt{2} \\ \sqrt{8} + \sqrt{18} &= 2\sqrt{2} + 3\sqrt{2} \\ &= 5\sqrt{2} \quad \text{irrational}\end{aligned}$$

Multiplying Square Roots

Now consider multiplying square roots. Remember, if both factors are rational, the product is rational. If one factor is rational and one is irrational, the product is irrational (assuming the rational factor isn't zero). Consider two more examples.

Example 4: Simplify $\sqrt{4} \times \sqrt{9}$. Is this product rational or irrational?

You should recognize that 4 and 9 are perfect squares, so both can be simplified as rational numbers. Therefore, the product will be rational. If you aren't sure, factor. You should be able to do the math for this one in your head, but if not, write out the steps.

$$\sqrt{4} \cdot \sqrt{9} = 2 \cdot 3 = 6 \quad \text{rational}$$

Example 5: Simplify $\sqrt{16} \times \sqrt{8}$. Is this product rational or irrational?

You should recognize that 16 is a perfect square but 8 is not; therefore, this problem represents a product of a rational and an irrational number. The product is irrational, but this expression can still be simplified.

Remember, the multiplied terms can be regrouped because of the associative property. Multiply the integers to simplify.

$$\begin{aligned}\sqrt{16} &\longrightarrow 4 \\ \sqrt{8} &\longrightarrow 2\sqrt{2} \\ \sqrt{16} \cdot \sqrt{8} &= 4 \cdot 2\sqrt{2} \\ &= 4 \cdot 2 \cdot \sqrt{2} = 8\sqrt{2} \\ &\quad \text{irrational}\end{aligned}$$

Adding two irrational numbers is always irrational. Is that also true for multiplying two irrational numbers? No, there is no rule for multiplying two irrational numbers because the product can be either rational or irrational. Consider a few more examples.

Section 1.6, continued

Rational vs. Irrational Numbers

Example 6: Simplify $\sqrt{18} \times \sqrt{2}$. Is this product rational or irrational?

Since neither 18 nor 2 are perfect squares, you should recognize this problem as a product of two irrational numbers. Can you determine if the product is rational or irrational? No, you cannot assume without doing the math.

First, separate everything under the radicals into prime factors. The 2 is already prime, but the 18 becomes $3 \times 3 \times 2$.

Using the associative property and the multiplication property of roots, these factors can be combined under one radical or can be rearranged and rewritten as $(\sqrt{3 \cdot 3})(\sqrt{2 \cdot 2})$.

The end product is 6, a rational number.

$$\sqrt{18} \cdot \sqrt{2} = \sqrt{3 \cdot 3 \cdot 2} \cdot \sqrt{2}$$

$$= \sqrt{3 \cdot 3 \cdot 2 \cdot 2}$$

$$= \sqrt{3 \cdot 3} \cdot \sqrt{2 \cdot 2}$$

$$= 3 \cdot 2 = 6 \quad \text{rational}$$

Example 7: Simplify $\sqrt{12} \times \sqrt{32}$. Is this product rational or irrational?

This is another example of two irrational numbers being multiplied together. First, separate each radical into prime factors. Then, instead of rewriting all those 2's under a single radical, let's simplify each separately. Feel free to take a shortcut when simplifying once you see what you have. In this example, we've skipped some steps. Can you see what we did?

When each square root term is simplified separately, the expression becomes $2\sqrt{3} \times 4\sqrt{2}$.

Using the commutative and associative properties, the multiplied terms can be rearranged and regrouped. Using the multiplication property of roots, the square roots can be combined under a single radical.

The numbers outside the radical can be multiplied together, and the numbers under the radical can be multiplied together. The final simplified product, $8\sqrt{6}$, is an irrational number.

Could you have rewritten everything under a single radical sign? Sure! Try it. You should get the same answer.

$$\sqrt{12} \rightarrow \sqrt{2 \cdot 2 \cdot 3} = 2\sqrt{3}$$

$$\sqrt{32} \rightarrow \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = 4\sqrt{2}$$

$$2\sqrt{3} \cdot 4\sqrt{2}$$

$$= 2 \cdot 4 \cdot \sqrt{3} \cdot \sqrt{2}$$

$$= 2 \cdot 4 \cdot \sqrt{3 \cdot 2}$$

$$= 8\sqrt{6} \quad \text{irrational}$$

Squaring and Cubing Square Roots

What happens when you square a square root? By now, you should recognize that squaring a square root eliminates the radical sign. Therefore, squaring the square root of a whole number will always result in a rational number.

Raising a square root to any even number power will eliminate the radical sign. Raising a square root to an odd number power, however, will not. For example, cubing a square root does not automatically eliminate a radical sign.

Examples of Squaring Square Roots

$$\frac{(\sqrt{2})^2 = 2}{(\sqrt{0.5})^2 = 0.5} \quad \left| \quad \left(\sqrt{\frac{3}{7}}\right)^2 = \frac{3}{7}\right.$$

Relations and Functions

Section 6.4 Function Notation



Key Term 6.4

- **Function notation** – a shorthand way of indicating that one variable in an equation is a function of another variable; often uses $f(x)$ instead of y to indicate that the relationship between the two variables is a function of x

So far you've seen sets of ordered pairs that represent functions, but functions are more often represented by equations in two variables. When an equation contains two variables, the equation is a function if one of the variables depends on the other variable. In terms of x and y , an equation is a function if the value of y is dependent on the value of x .

When one variable is a function of the other variable, the equation can be written in a type of shorthand called **function notation**. Consider the variables of x and y . Remember that x values represent independent values in the domain and y values represent dependent values in the range. In an equation, if y is a function of x , the y can be replaced with "function of x ." The shorthand notation is often shown as $f(x)$, which is read as " f of x ." The function notation goes on the left side of the equals sign. The right side of the equal sign is sometimes called the "rule."

Be careful that you don't make the mistake of thinking that $f(x)$ means $f \cdot x$. It might be easy to make that mistake because you've gotten used to multiplying anything in parentheses. However, the parentheses are part of the function notation.

$$y = 2x + 3$$

Consider the following equation that contains the variables of x and y . Can you see that the value of y depends on the value of x ? You can choose any value of x , substitute that value into the equation, and get a value for y . Therefore, y is a function of x .

$$f(x) = 2x + 3$$

Since y is a function of x , we can indicate the relationship by replacing the y with $f(x)$. The rule in this function is $2x + 3$.

$$f(x), g(x), h(x), k(x)$$

The letter " f " is not the only one used to denote a function. The letters g , h , and k are also commonly used to show a function, but almost any letter can be used.

Evaluating a Function

When a function is written in function notation, you can substitute values for the domain (x) into the rule to get a corresponding value for range (y). Solving a function for a given value is called "evaluating the function" for that value.

$$f(x) = 2x + 3$$

Remember, the values of x in this function are the domain values. If you substitute a value for x , you can get the corresponding range value. What is the range value when $x = 2$?

$$f(2) = 2(2) + 3$$

To show that you are evaluating the function when $x = 2$, you replace x with 2. The notation now means the "functional value of f when $x = 2$." When solved, the functional value, or range value, is 7.

$$f(2) = 7$$

Functions are not always written in terms of x and y . They can be written with other variables as well. For example, consider the equation $c = 2a + 5$. The c is the dependent variable, and the a is the independent variable. This equation could be rewritten as $f(a) = 2a + 5$ to indicate that c is actually a function of a . In this case, you could substitute any value of a into the equation to find the dependent value, $f(a)$.

Arithmetic Sequences

Section 10.2

The Arithmetic Sequence Formula



Once you've determined that a list of numbers is an arithmetic sequence, you can easily determine the next term in the sequence. As you just saw, simply add or subtract the common difference. But what if you want to find the 50th term in the sequence? There's a formula for that!

The formula for finding any term in an arithmetic sequence is given below. It may look a little complicated, but once you understand what each term stands for, it isn't difficult to use.

Arithmetic Sequence Formula

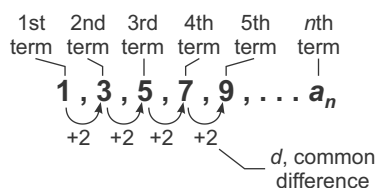
$$a_n = a_1 + (n - 1)d$$

a_n is the n th term in the sequence

a_1 is the 1st term in the sequence

n is term number

d is the common difference



Example 1: In the following arithmetic sequence, find the 25th term.

$$3, 5, 7, 9, 11, \dots$$

To find any term in this sequence, all you need is the formula and the common difference. In the given sequence, the first 5 terms are listed. You can easily see that the common difference is 2. To get the 25th term, you could keep adding 2 until you get to the 25th term, or you can use the formula. Let's use the formula!

To get the 25th term, a_{25} , the n is 25. From the sequence, determine a_1 and d . It may be helpful if you write what each term represents for the specific sequence. The first term, a_1 , is 3. The common difference is easily calculated as 2. If you aren't sure, simply subtract the second term from the first, $5 - 3 = 2$.

$$a_n = a_{25} \quad a_1 = 3$$

$$n = 25 \quad d = 2$$

Substitute these values into the formula and solve. When you do the math, you get the 25th term as 51.

$$a_{25} = 3 + (25 - 1)2$$

$$a_{25} = 51$$

Example 2: In the following arithmetic sequence, find the 10th term.

$$0, -3, -6, -9, \dots$$

In this case, the values in the sequence are getting smaller, so the common difference is negative. In other words, the same number is being subtracted from each term. Work this type of problem the same way, but be sure to use a negative value for d .

In this problem, n is 10, a_1 is 0, and the common difference, d , is -3 .

$$a_n = a_{10} \quad a_1 = 0$$

$$n = 10 \quad d = -3$$

Substitute these values into the formula, and do the math.

$$a_{10} = 0 + (10 - 1)(-3)$$

You get a_{10} , the 10th term, as -27 .

$$a_{10} = -27$$

Exponential Function Applications

Section 15.2 Exponential Growth and Decay as Percentages



If a population doubles, the b value is 2. If it triples, the b value is 3. If it halves, the b value is 0.5. These values for b are easy to determine. However, the rate of exponential growth or decay is often given as a percentage instead of as an even multiple. For example, a population in a city may be increasing by 5% each year, or the value of your car may decrease by 15% for each year that you own the car. These types of situations are modeled by the same exponential growth/decay function BUT the b value must be converted to a decimal equivalent. Let's take a closer look at these types of real-world exponential scenarios.

Exponential Growth

Let's consider a population that doubles over a set interval. You've already seen that the b value is 2. What percent increase is the population experiencing? In terms of a percentage, the population is increasing by 100%. To better understand, think in terms of money. Let's say you have \$100 in savings, and your rich uncle says he will match your savings by a percentage. If he matches your \$100 by 50%, he is adding \$50, right? 50% is the same as one-half. What if he matches 100%? Then he is adding another \$100, and your money is doubled. When something increases by 100%, it doubles.

If doubling a population is equivalent to a 100% increase, what would it mean if the population is increasing by less than double? The b value will then be less than 2 but still greater than 1.

In many cases, the growth is less than double, so it is given as a percentage. If you are given exponential growth as a percentage, convert the percentage to a decimal by dividing by 100 and then add 1. This value becomes the b value in the exponential growth formula. This "percent growth" formula is given to the right. The r value is the percent increase given as a decimal number.

Exponential Growth/Decay

$f(x) = a \cdot b^x$

amount at given interval
initial amount
number of intervals
common ratio

percent increase: $b = (1 + r)$
 percent decrease: $b = (1 - r)$
 r is the decimal value of the given percentage

Percent Growth Formula

$f(x) = a \cdot (1 + r)^x$

where r is percent increase as a decimal

Example 1: The population of a certain city in Mississippi has been steadily increasing at 6% per year. What is the common ratio (the value of b)?

How do you represent 6% as a common ratio? First, divide 6 by 100. In other words, move the decimal two places to the left. Then add 1. In this problem, r is 0.06, and the b value is 1.06. Notice that this value is less than 2 but greater than 1, which is exactly what you would expect.

$$r = 6\% = 0.06$$

$$b = 1 + r$$

$$b = 1 + 0.06 = 1.06$$

Section 15.2, continued
Exponential Growth and Decay
as Percentages

Example 2: If the initial population in Example 1 is 50,000, write a function to represent the growth. What is the population after 2 years?

You work this problem the same as you did in Section 15.1. The interval, x , is per year. The initial value, a , is 50,000. You've already calculated b , the common ratio, as 1.06.

Let x = per year

$$f(x) = 50,000(1.06)^x$$

To find the population after 2 years, evaluate the function for $x = 2$. Use a calculator to do the math, and you get a population of 56,180 after 2 years.

$$f(2) = 50,000(1.06)^2$$

$$f(2) = 56,180$$

Practice 1

For each word problem, determine the interval, x , the initial value, a , the percent increase, r , and the common ratio, b . Use these values to write an exponential function, and then use the function to answer the question.

-
1. An initial population of 50 wild gorillas increases by 12% every 5 years.

The interval, x , represents _____. a is _____. r is _____. b is _____.

The function is _____.

What will the population of gorillas be after 5 years? _____

-
2. An employee with a starting salary of \$55,000 per year is given a raise of 3% per year.

The interval, x , represents _____. a is _____. r is _____. b is _____.

The function is _____.

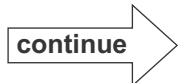
What will the employee's yearly salary be in 5 years? Round to the nearest penny. _____

-
3. One pair of breeding rabbits is introduced into the ecosystem of an island with no other rabbits. The rabbit population increases by 80% each month. (Hint: one breeding pair means 2 rabbits, a male and a female.)

The interval, x , represents _____. a is _____. r is _____. b is _____.

The function is _____.

How many rabbits will inhabit the island in 1 year? Round to the nearest whole number. _____



Factoring Polynomials

Section 18.5 Equivalent Expressions



Polynomials may be expressed in different forms. They can be completely simplified, they can be factored into prime factors, or they can be partially factored. It's important that you be able to recognize equivalent polynomials. The easiest way to determine equivalent polynomials may be to simplify and compare, but you may be able to recognize others without having to simplify completely. Consider the following.

How many ways can the following polynomial be rewritten as an equivalent expression?

$$4x^2 - 9y^2$$

Do you recognize this polynomial as a difference of squares?

$$(2x)^2 - (3y)^2$$

1. Before we actually do the factoring, let's rewrite it as a square of the two terms. Can you see that this rewritten polynomials is equivalent to the one above?

$$(2x - 3y)(2x + 3y)$$

2. Now factor as a difference of squares. In this form, the polynomial is completely factored. These two binomial factors represent an equivalent expression to the original.

What happens when you have terms raised to a power of 4?

$$16x^4 - 81$$

You should recognize this polynomial also as a difference of squares. Let's find equivalent expressions.

$$(4x^2)^2 - (9)^2$$

1. First, let's rewrite it as a square of the two terms as we did before. This binomial expression is equivalent to the one above it.

$$[(2x)^2]^2 - [(3)^2]^2$$

2. Notice that the $4x^2$ and the 9 are still perfect squares. Can we rewrite these again as squares? Sure we can! This one may look a little ugly, but it's still equivalent to the others.

$$(2x)^4 - (3)^4$$

3. Using the rule of raising a power to a power, could we rewrite this expression again? Let's do it. Remember, when you raise a power to a power, you multiply the exponents. Now we have yet another equivalent expression.

$$(4x^2 - 9)(4x^2 + 9)$$

4. Now let's rewrite as binomial factors. Let's begin with the original expression and rewrite it as a sum and difference of terms.

$$(2x - 3)(2x + 3)(4x^2 + 9)$$

5. Can these binomial factors be further factored? You should recognize that the binomial $(4x^2 - 9)$ is also a difference of squares. Let's factor it as a sum and difference of terms. This expression is also equivalent to the original. Notice that the $(4x^2 + 9)$ cannot be further factored. Remember, you cannot factor a sum of squares.

As you should be able to see from the above examples, factors of a polynomial do not have to be prime in order to still be factors. Different combinations of prime and not prime factors can make up equivalent expressions. Let's consider one more example.

Section 18.5, continued

Equivalent Expressions

Example: Find all the possible factors of $4x^2 + 20x + 24$.

This example says to find all possible factors. Use what you already know to find some of the factors. Then consider some other ways to find additional ones.

1. First, factor out the common term of 4. This step gives you two factors of this expression, 4 and $(x^2 + 5x + 6)$. Neither of these is prime, but they are still factors, and this is an equivalent expression.

$$\frac{(4)(x^2 + 5x + 6)}{\uparrow \quad \uparrow} \quad 2 \text{ factors}$$

2. Next, factor the trinomial into two binomial factors by using trial and error. Now you have three factors for the original expression and another equivalent expression.

$$\frac{(4)(x + 3)(x + 2)}{\uparrow \quad \uparrow \quad \uparrow} \quad 3 \text{ factors}$$

3. The two binomials are prime since they cannot be factored further. But the factor of 4 isn't prime. It can be factored into 2 times 2. The prime factors are 2, 2, $(x + 3)$, and $(x + 2)$.

$$\frac{(2)(2)(x + 3)(x + 2)}{\quad \quad \quad \quad} \quad 4 \text{ prime factors}$$

Each of these factored expressions are equivalent. Is it possible to have other factors and to have additional equivalent expressions? Consider one more step.

4. Now that you have the prime factors, you can still find other possible factors — factors that are not prime — and additional equivalent expressions. By multiplying either of the binomials by one or both of the other prime factors, you get additional factors. There are quite a few possibilities. Below, see how you can use the distributive property to find the other possible factors and additional equivalent expressions.

| | |
|---|--|
| $(2)(2)(x + 3)(x + 2) \longrightarrow (2)(2x + 6)(x + 2)$ | $(4)(x + 3)(x + 2) \longrightarrow (4x + 12)(x + 2)$ |
| $(2)(2)(x + 3)(x + 2) \longrightarrow (2)(x + 3)(2x + 4)$ | $(4)(x + 3)(x + 2) \longrightarrow (x + 3)(4x + 8)$ |
| $(2)(2)(x + 3)(x + 2) \longrightarrow (2x + 6)(2x + 4)$ | |

From the above example, you can see how a single polynomial expression can have many equivalent expressions that use a combination of possible factors.

Practice 1

How many equivalent expressions can you write for each polynomial below? List as many as you can by finding as many possible factors as you can. (There may be more possibilities than blanks.)

1. $6x^2 - 18x + 12$

| | |
|--|--|
| | |
| | |
| | |
| | |

2. $9x^2 + 45x + 36$

| | |
|--|--|
| | |
| | |
| | |
| | |

Transformations of Functions

Section 28.1 Transformation Basics



Key Terms 28.1

- **Dilation** – a transformation that compresses or stretches a line or a curve; for any function $f(x)$, $f(Cx)$ causes a horizontal dilation and $Cf(x)$ causes a vertical dilation
- **Reflection** – a transformation that creates a mirror image of a line or curve across an axis of symmetry; for any function $f(x)$, $f(-x)$ causes a horizontal reflection across the y -axis and $-f(x)$ causes a vertical reflection across the x -axis
- **Transformation** – a change in the shape or position of a graphed line or curve
- **Translation** – a type of transformation that changes the vertical and/or horizontal position of a graphed line or curve; for any function $f(x)$, $f(x + C)$ causes a horizontal translation and $f(x) + C$ causes a vertical translation

In Section 26, you learned about performing operations on functions, and you saw how different operations were written using function notation. Now let's relate those operations on functions to the graphs of a line or curve.

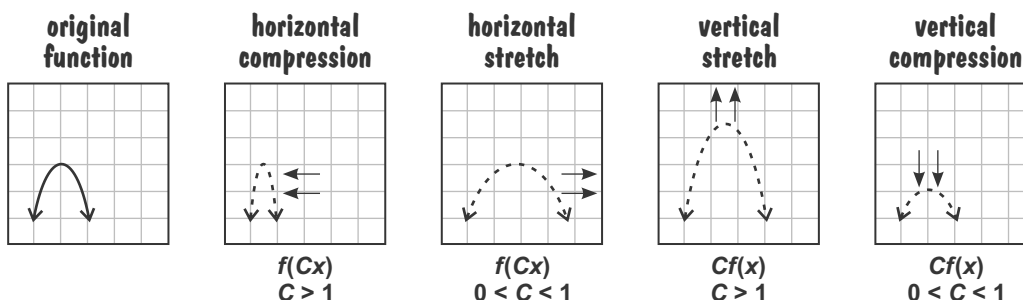
Any operation on a function results in a change in either the shape or the position of the graphed line or curve. This change is called a **transformation**. Since you have already reviewed the basics of different types of functions and their graphs, transformations shouldn't be difficult. You may even find them interesting. Let's go over the basics first, and then we'll look at each type of transformation for specific types of functions.

Dilations

A **dilation** is a type of transformation that compresses or stretches a line or a curve. There are two types of dilations: horizontal and vertical.

- A *horizontal dilation* changes the shape of a function from side to side along the x -axis. Think of a horizontal dilation as taking a graph and squeezing it inward from the sides to compress it or pulling it outward from the sides to stretch it. For any function $f(x)$, the operation that causes a horizontal dilation is $f(Cx)$, where C represents any number that is multiplied to the variable.
- A vertical dilation changes the shape of a function from top to bottom along the y -axis. Think of a *vertical dilation* as taking a graph and squeezing it from the top and bottom to compress it or pulling it from top and bottom to stretch it. For any function $f(x)$, the operation that causes a vertical dilation is $Cf(x)$, where C represents any number that is multiplied to the function.

The different dilations are summarized by the graphs below. Notice that when C is greater than 1, the function compresses horizontally for $f(Cx)$ but stretches vertically for $Cf(x)$. However, when C is a fraction or decimal value between 0 and 1, the function stretches horizontally for $f(Cx)$ but compresses vertically for $Cf(x)$. You will see specific examples of each of these behaviors in Section 28.2. For now, simply be aware of the general rules.



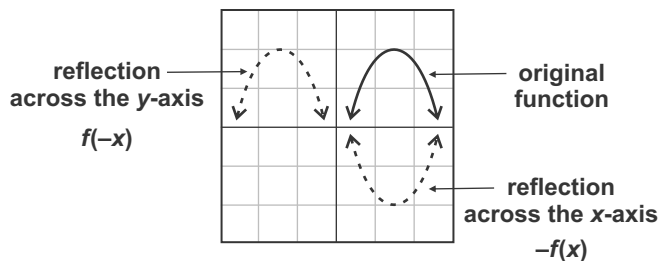
Section 28.1, continued

Transformation Basics

Reflections

A **reflection** is a type of transformation that creates a mirror image of a function across an axis of symmetry. A reflection doesn't change the shape of the function, but it changes its orientation. Two types of reflections include reflections across the y -axis and reflections across the x -axis.

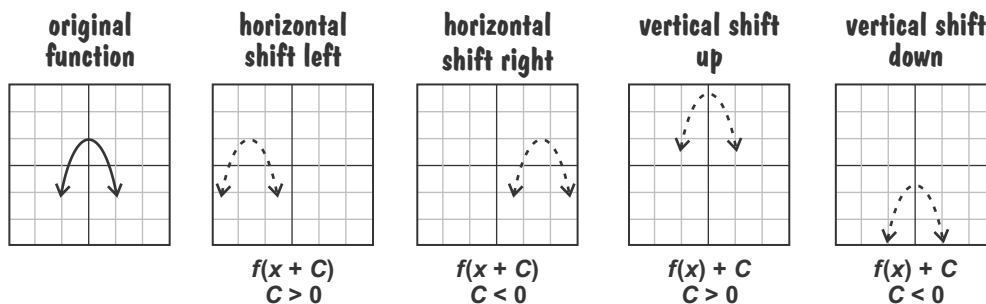
- For any function $f(x)$, the operation that causes a horizontal reflection across the y -axis is $f(-x)$.
- For any function $f(x)$, the operation that causes a vertical reflection across the x -axis is $-f(x)$.



Translations

A third type of transformation, called a **translation**, shifts the position of a function. The shape of the function doesn't change. Only its position on the graph changes. There are two types of translations: horizontal and vertical.

- A *horizontal translation* shifts the position of a graphed line or curve left or right. For any function $f(x)$, the operation that causes a vertical shift is $f(x + C)$, where C represents any number that is added to the variable. When C is a positive value (greater than 0), the function shifts left. When C is a negative value (less than 0), the function shifts right. (These shifts may be opposite of what you would assume, but you will better understand why functions shift this way once you review Section 28.4. Memorizing these behaviors is also helpful!)
- A *vertical translation* shifts the position of a graphed line or curve up or down. For any function $f(x)$, the operation that causes a vertical shift is $f(x) + C$, where C represents any number that is added to the function. When C is a positive value, the function shifts up, but when C is a negative value, the function shifts down.



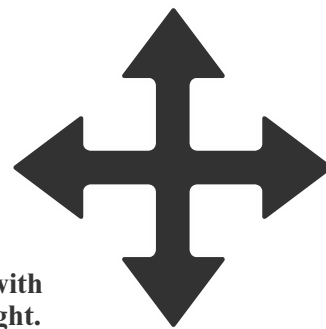
Practice

Circle the term in each set of parentheses that correctly completes the sentence.

1. A transformation that changes the shape of a function is a (dilation / translation).
2. A (horizontal / vertical) compression changes the shape of a function along the x -axis.
3. A (horizontal / vertical) stretch changes the shape of a function along the y -axis.
4. A transformation that creates a mirror image across an axis of symmetry is called a (dilation / reflection).
5. A transformation that changes the position of a function on the graph is called a (reflection / translation).
6. A transformation that shifts a function left on a graph is a (horizontal / vertical) translation.
7. A transformation that shifts a function down on a graph is a (horizontal / vertical) translation.

Transformations of Functions

Section 28.5 Vertical and Horizontal Translations



A translation of a function is not limited to a single shift. A function can be shifted both horizontally and vertically. Consider the following examples.

Example 1: Write a function $g(x)$ that represents the function $f(x) = 2x - 1$ with a vertical shift 2 units up and a horizontal shift 3 units to the right.

This example is asking you to take a function $f(x)$ and translate it 2 units up and 3 units right. The new function will be designated as $g(x)$.

To shift a line up 2 units, you add to the function, $f(x) + 2$. To shift a line to the right 3 units, you subtract from the variable, $f(x - 3)$. To get $g(x)$, simply perform both of these shifts. (The order doesn't matter.)

$$g(x) = f(x - 3) + 2$$

Starting with the original function $f(x)$, add 2 to get the vertical shift and substitute $(x - 3)$ for x to get the horizontal shift. Do the math to simplify. The new function becomes $g(x) = 2x - 5$.

$$g(x) = 2(x - 3) - 1 + 2$$

$$g(x) = 2x - 6 - 1 + 2$$

$$g(x) = 2x - 5$$

Example 2: Given the function $f(x) = (x + 2)^2$, write a function $g(x)$ that represents $f(x)$ with a vertical shift 1 unit down and a horizontal shift 2 units to the left.

This example is asking you to take a function $f(x)$ and translate it 1 unit down and 2 units left. The new function will be designated as $g(x)$.

Shifting a quadratic function in vertex form is easy. To better see this one in vertex form, add a zero to represent the k value.

$$f(x) = (x + 2)^2 + 0$$

To shift the function down 1 unit, use $f(x) - 1$. To shift the function to the left 2 units, use $f(x + 2)$. To get $g(x)$, perform both of these shifts.

$$g(x) = f(x + 2) - 1$$

$$g(x) = (x + 2 + 2)^2 - 1$$

Notice that you add 2 to the h constant inside the parentheses and you subtract 1 from the k constant outside the parentheses.

$$g(x) = (x + 4)^2 - 1$$

Example 3: Given the function $f(x) = 5|x|$, write a function $g(x)$ that represents $f(x)$ with a vertical shift 3 units up and a horizontal shift 5 units to the left.

This example is asking you to take a function $f(x)$ and translate it 3 units up and 5 units left. Designate the new function as $g(x)$.

Shifting an absolute value function is also simple, but you need to recognize that the h and k values of $f(x)$ are both zero. You may want to add the zeros to better see the vertex form.

$$f(x) = 5|x - 0| + 0$$

To shift the function up 3 units, use $f(x) + 3$. To shift the function to the left 5 units, use $f(x + 5)$. In other words, add 5 to the value of h and add 3 to value of k . The vertex moves from $(0, 0)$ to $(-5, 3)$, so you can see that $g(x)$ moves to the left and up.

$$g(x) = f(x + 5) + 3$$

$$g(x) = 5|x + 5| + 3$$

Measures of Center

Section 30.2

Dot Plots



Key Terms 30.2

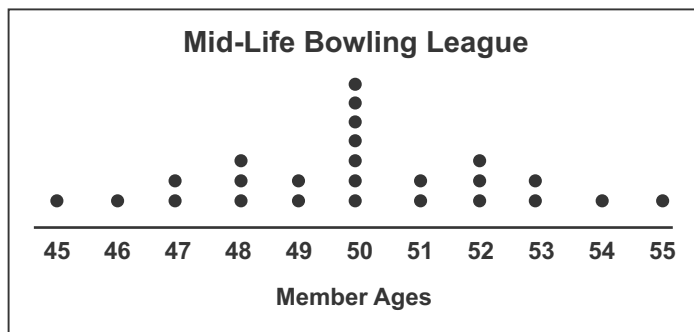
- **Discrete data** – data that is counted with whole numbers
- **Dot plot** – a statistical diagram used to display each piece of data as a dot over a number line
- **Skewed** – used to describe data that is not distributed symmetrically around the mean so that mean, median, and mode are different values
- **Symmetric** – used to describe data that is evenly distributed around the mean so that mean, median, and mode are the same value

Measures of central tendency give single values that tell something important about a set of values. Another way to present data in a meaningful way is to display it as a diagram. Two types of statistical diagrams help to give measures of central tendency additional context: dot plots and histograms. Let's look at dot plots first.

Dot Plots

A **dot plot** is a type of statistical diagram that uses dots to represent data points. The dots are displayed over a number line. The dots that represent the same value are stacked on top of one another. (Don't confuse dot plots with scatter plots. Dot plots are used to display a single variable.) Dot plots are most often used to display **discrete data**, which is data that is counted using whole numbers. For example, the number of lemons on a lemon tree is discrete data. The number of lemons will always be a whole number and not a fraction or decimal number.

Look at the dot plot below. The variable being shown is the age of members in a bowling league. Each dot represents the age of a member in the league.



How would you represent this data as a set of numbers? For each dot, record the age under it. As a set, you have $\{45, 46, 47, 47, 48, 48, 48, 49, 49, 50, 50, 50, 50, 50, 50, 50, 50, 51, 51, 52, 52, 52, 53, 53, 54, 55\}$.

Which is easier to interpret — the list or the dot plot? Most would say the dot plot! Consider the following example.

Example 1: What are the mean, median, and mode of the data shown in the dot plot of the Mid-Life Bowling League?

Before doing any calculations, what are your best guesses for mean, median, and mode? Are some of these values easy to identify on the graph? If so, which ones?

Section 30.2, continued

Dot Plots

- To get the actual mean, you need to add all the values in the set and divide that sum by the number of dots. The sum of the members' ages is 1250. The number of dots (members) is 25. Therefore the mean is 50.

$$\text{mean} = \frac{1250}{25} = 50$$

- To determine the median, you want the point in the middle. If there are 25 points, the middle is the 13th data point (12 before it and 12 after it). If you simply count points from either direction, the 13th point falls in the 50 column. Therefore, the median value is 50.

$$\text{median} = 50$$

- On a dot plot, the mode is easy to identify. It'll be the column (or columns) with the most dots. The mode is also 50 because 7 members in the league are 50 years old. The league has more members who are 50 than any other specific age.

$$\text{mode} = 50$$

Symmetric Data versus Skewed Data

Notice that for the Mid-Life Bowling League data, the mean, the median, and the mode are all the same. When data is evenly distributed around a center value, the data is said to be **symmetric**. In symmetric data, the mean, median, and mode will be the same or nearly the same. Look again at the dot plot for the Mid-Life Bowling League. Notice that the points on either side of 50 are mirror images. (Data points do not have to be perfect mirror images to be symmetric, but they will be close.)

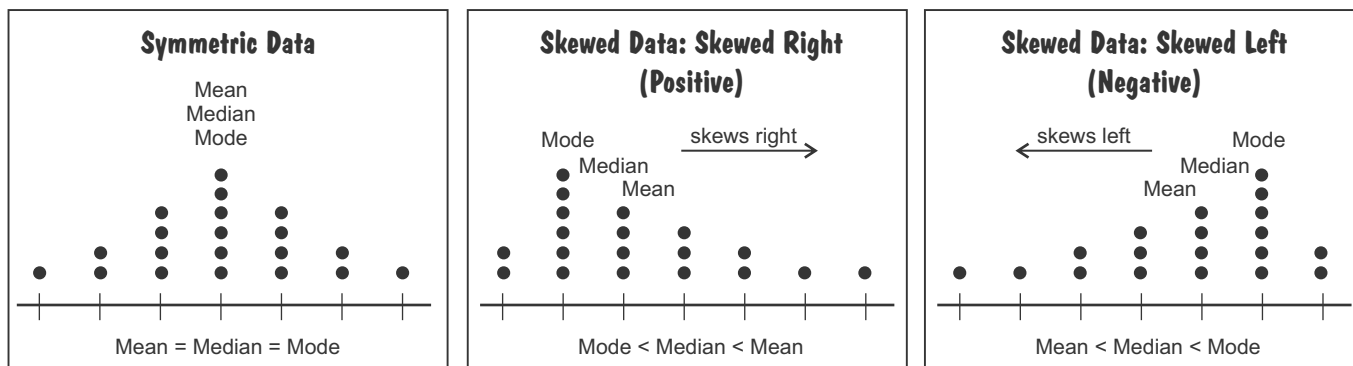
Several types of real-world data tend to be symmetric. For example, a standardized test often has symmetric scores. Most of the scores are in the middle; the rest of the scores are fairly evenly distributed on either side of the middle.

Other types of real-world data are not symmetric but **skewed**. Skewed data has different values for mean, median, and mode. Data that is not symmetric will either skew to the right or to the left.

- When data is skewed to the right, the skew is also considered to be positive. In a positive skew, the mode is to the *left* of the middle, which means the highest column of data is on the left. The median is to the right of the mode, and the mean is to the right of the median. You can think of this skew as the median and mean being “pulled” to the right. A longer “tail” of the data will be on the right side.
- When data is skewed to the left, it is said to have a negative skew. For a negative skew, the mode is to the *right* of the middle. The median is to the left of the mode, and the mean is further left. You can think of this skew as the median and mean being “pulled” to the left. A longer “tail” of the data will be on the left side.

(These trends hold true as long as the data has only one mode.)

The types of data are summarized in the diagrams below.



Categorical Data

Section 32.2 Two-Way Frequency Tables



Key Terms 32.2

- **Joint frequency** – a value in the middle of a two-way frequency table that represents the number of items that satisfy both variables represented by the row and the column
- **Marginal frequency** – a value on the far right or bottom of a two-way frequency table that is the sum of the items in that column or row
- **Two-way frequency table** – a table that shows data for two different categorical variables; items in each row and column of the table satisfy a category in each of the two different variables

A **two-way frequency table** shows data for two different categorical variables instead of for just one variable. The items in each row and column of the table satisfy a category in each of the two different variables. Two-way frequency tables are used to compare and analyze relationships between two variables and among the categories of the variables. Let's look at an example.

Let's say a science class studying genetics surveys 150 ninth-grade students. The science class records eye color and hair type for each of the survey participants. The table below organizes the survey data into a two-way frequency table.

Student Eye Color vs. Hair Type

| | blue | brown | green | hazel | Total |
|----------|------|-------|-------|-------|-------|
| straight | 7 | 17 | 13 | 6 | 43 |
| curly | 5 | 43 | 3 | 14 | 65 |
| wavy | 6 | 19 | 8 | 9 | 42 |
| Total | 18 | 79 | 24 | 29 | 150 |

The two variables being compared in this table are eye color and hair type. Eye colors are listed across the top, and hair types are listed on the left.

Two-way frequency tables give two types of data: joint frequencies and marginal frequencies.

A **joint frequency** is one that appears in the middle of the table. Each joint frequency gives the number of items (or students in this case) that satisfy a category in each of the variables. The joint frequencies are shown in the lighter gray boxes in the table. What is the joint frequency for blue eyes and curly hair? That joint frequency is 5, meaning 5 of the students have these two characteristics. The most common joint frequency is the one with the greatest value. In this table, the most common joint frequency is brown eyes with curly hair, which is true for 43 students. The least common joint frequency is green eyes with curly hair; only 3 students have this combination of characteristics.

A **marginal frequency** is a sum of a row or a column. In other words, it represents a total for each row and column. The marginal frequencies are found in the margins, which are in the bottom row and in the last column on the right. In the table above, the marginal frequencies are in the darker gray boxes. In the Student Eye Color vs. Hair Type survey, the total number of students with blue eyes is 18. The total number of students with wavy hair is 42. These two values are marginal frequencies. What is the marginal frequency for hazel eyes? That value is 29, the sum of the hazel column, so 29 students have hazel eyes. What is the marginal frequency of curly hair? That value is 65, the sum of the curly row, so 65 students have curly hair.

The number in the bottom right corner is the total number of items represented by the table. The sum of the marginal frequencies in the right column will equal the sum of the marginal frequencies in the last row. Both of these will equal to total number of items in the table. For the Student Eye Color vs. Hair Type data, notice that the sum of the marginal frequencies of the last column ($43 + 65 + 42$) equals 150 and the sum of the marginal frequencies of the bottom row ($18 + 79 + 24 + 29$) also equals 150.

Categorical Data

Section 32.4

Conditional Relative Frequencies



Key Term 32.4

- **Conditional relative frequency** (or **conditional probability**) – a joint frequency divided by one of its corresponding marginal frequencies

A two-way relative frequency table gives percentages of the whole, but it does not give percentages of one category compared to just one of the variables. Look again at the frequency table for college freshmen and their favorite sport to play. This time, consider both the two-way frequency table and a two-way relative frequency table.

Favorite Sport to Play

| | Basketball | Tennis | Soccer | Total |
|-------|------------|--------|--------|-------|
| Men | 84 | 36 | 55 | 175 |
| Women | 27 | 29 | 19 | 75 |
| Total | 111 | 65 | 74 | 250 |

Two-Way Frequency Table

Favorite Sport to Play

| | Basketball | Tennis | Soccer | Total |
|-------|------------|--------|--------|-------|
| Men | 0.336 | 0.144 | 0.22 | 0.7 |
| Women | 0.108 | 0.116 | 0.076 | 0.3 |
| Total | 0.444 | 0.26 | 0.296 | 1 |

Two-Way Relative Frequency Table

Carefully consider the information given by the two-way relative frequency table on the right. The marginal frequencies tells us that 70% of all the freshmen who were surveyed are men and 30% are women. It tells us that 44.4% of the men and women prefer basketball, 26% prefer tennis, and 29.6% prefer soccer. Now consider one of the joint frequencies. The table tells us 33.6% of all those surveyed are men who prefer basketball.

Now consider what these tables do NOT tell us. Do we know from the relative frequencies how many men prefer basketball given just the men? You may be tempted to say 33.6%, but that percentage considers *both men and women*. To get the percentage of just the men who prefer basketball, we need a different calculation. Looking at the two-way frequency table, we would need to divide the number of men who prefer basketball, 84, by the total number of men, 175 (not 250 that was used to get the joint relative frequency). If you do the math, you get 0.48. In other words, 48% of just the men who were surveyed prefer basketball.

A **conditional relative frequency** is calculated by dividing a joint frequency by one of its corresponding marginal frequencies. A conditional relative frequency can also be called a **conditional probability**. A conditional relative frequency is what we just calculated to determine the number of just the men who prefer basketball.

Conditional Relative Frequency

$$\frac{\text{joint frequency}}{\text{marginal frequency}}$$

A conditional relative frequency can be calculated using either of the corresponding marginal frequencies, but the resulting values will represent something different. For example, let's take the joint frequency of 84 that represents the number of men who prefer basketball and divide it by the marginal frequency of 111. Now you are dividing the number of men who prefer basketball by the total number of people who prefer basketball. The resulting value rounded to the nearest thousandth is 0.757, so 75.7% of just those who prefer basketball are men.

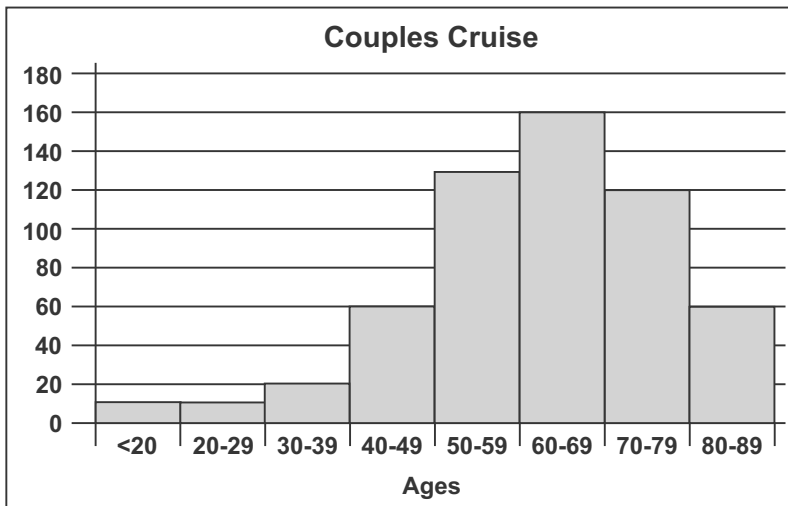
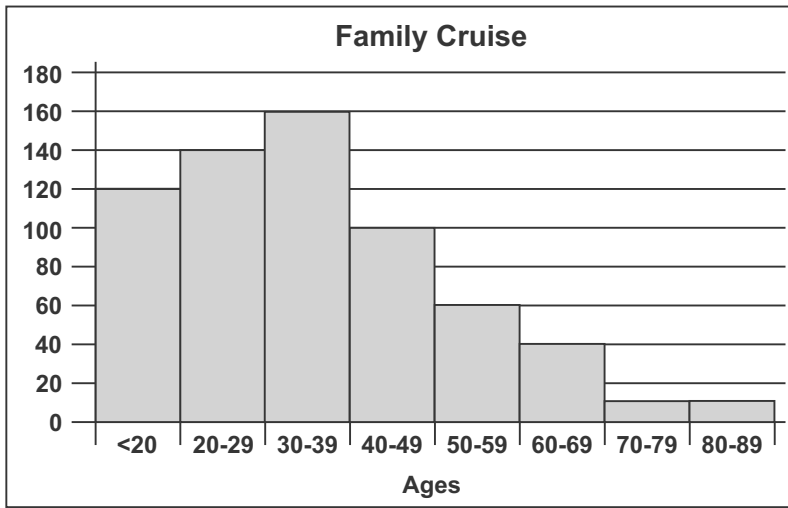
The Number of Men Who Prefer Basketball (considering just the men)

$$\frac{\text{the men who prefer basketball}}{\text{the total number of just men}} = \frac{84}{175} = 0.48 \text{ or } 48\%$$

The Number of Men Who Prefer Basketball (considering just those who prefer basketball)

$$\frac{\text{the men who prefer basketball}}{\text{the total number of just those who prefer basketball}} = \frac{84}{111} = 0.757 \text{ or } 75.7\%$$

18. A small cruise company offers two themed cruises the first week in July, one geared toward families and one for couples. Two histograms display the ages of the passengers who go on each of these cruises.



Which statements correctly interpret the data?

Select **two** answer choices.

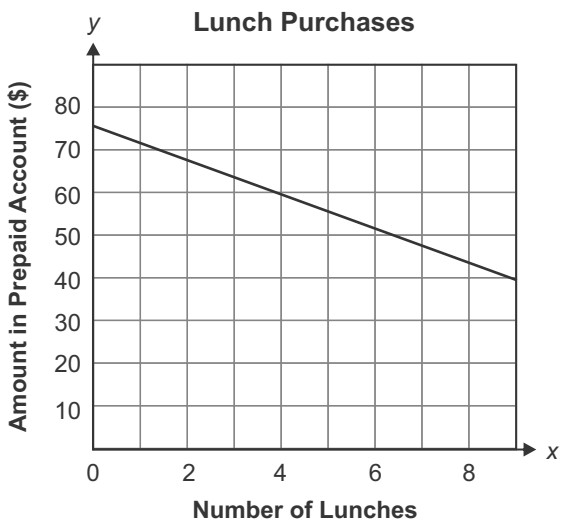
- (A) The family cruise has more passengers under 20 than the couples cruise has passengers over 79.
- (B) The median number of passengers on the family cruise is in the interval of 50-59.
- (C) The median number of passengers on the couples cruise is in the interval of 60-69.
- (D) The couples cruise has more passengers than the family cruise.
- (E) The data for the family cruise is skewed, but the data for the couples cruise is symmetric.



42. Select the box or boxes that represent the transformation of each function from the parent function $f(x) = |x|$.

| | Horizontal Dilation | Vertical Reflection | Vertical Translation | Horizontal Translation |
|----------------------|---------------------|---------------------|----------------------|------------------------|
| $f(x) = x - 2 + 2$ | (A) | (B) | (C) | (D) |
| $f(x) = 2 x $ | (A) | (B) | (C) | (D) |
| $f(x) = - x $ | (A) | (B) | (C) | (D) |
| $f(x) = x - 2 $ | (A) | (B) | (C) | (D) |

43. The function $T(m) = 75 - 4m$ represents the amount of money in Alice's prepaid school cafeteria account after she has purchased m lunches. The function is modeled by the graph.



Based on the scenario, which statement describes domain values of this function?

- (A) The domain of the function includes all real numbers.
- (B) The domain of the function includes all real numbers where $m \leq 75$.
- (C) The domain of the function includes all whole numbers where $m \geq 0$.
- (D) The domain of the function includes all whole numbers where $0 \leq m \leq 18$.



Algebra I

Practice Test 1

Evaluation Chart

Circle the questions you answered incorrectly on the chart below, and review the corresponding sections in the book. Read the instructional material, do the practice exercises, and take the section review tests at the end of each section.

| If you missed question #: | Go to section(s): | If you missed question #: | Go to section(s): | If you missed question #: | Go to section(s): |
|---------------------------|---|---------------------------|---|---------------------------|------------------------|
| 1 | 1.1, 1.2, 1.4, 1.5, 1.6 | 21 | 5.5 | 41 | 21.1, 21.2, 21.4, 23.1 |
| 2 | 11.1, 11.2, 11.3 | 22 | 21.1, 24.1, 24.2, 24.3 | 42 | Section 28 |
| 3 | 8.1, 11.1, 11.2, 11.4, 11.5 | 23 | 3.5 | 43 | 11.5, 11.6 |
| 4 | 1.2, 1.4, 2.1, 2.2, 2.3, 17.1, 17.3, 17.5, 17.7 | 24 | 3.1, 3.2, 3.3 | 44 | 12.1, 12.2 |
| 5 | 8.1 | 25 | 14.4, 14.5 | 45 | 27.1, 27.2, 27.4 |
| 6 | 29.3, 29.4, 29.5 | 26 | 6.4, 11.1, 11.2 | 46 | 4.1, 4.2, 4.3, 4.4 |
| 7 | 18.3, 19.1, 21.1, 21.2, 21.3 | 27 | 14.1, 14.3, 14.4 | 47 | 18.1, 18.4, 18.5 |
| 8 | 30.1, 30.2 | 28 | 6.4, 7.1 | 48 | 27.3 |
| 9 | 5.1 | 29 | 6.4, 11.1, 11.2, 11.3 | 49 | 32.1, 32.2, 32.3 |
| 10 | 17.1, 17.2, 17.3, 17.5, 17.6 | 30 | 6.4, 11.1, 11.2, 11.3, 26.1, 26.4, 28.1, 28.3 | 50 | 15.2 |
| 11 | 2.1, 2.2, 2.3, 17.1, 17.2, 17.3, 17.5, 17.7 | 31 | 13.1, 13.4 | 51 | 21.4, 24.1 |
| 12 | 1.7, 26.2 | 32 | 13.4, 13.5 | 52 | 23.1, 23.2 |
| 13 | 15.1, 15.2, 15.3, 15.4 | 33 | 8.2 | 53 | Section 10 |
| 14 | 4.1, 4.2, 4.3, 4.4, 4.6 | 34 | 4.1, 4.2, 4.4 | 54 | 25.1 |
| 15 | 3.5, 12.3, 13.7 | 35 | 8.1, 8.2, 9.1, 9.2, 9.3, 9.4 | 55 | 6.2, 6.4 |
| 16 | 21.1, 21.3 | 36 | 6.2, 6.3, 6.4, 7.4 | 56 | 23.3 |
| 17 | 13.2, 13.6 | 37 | 1.4, 17.1, 17.2, 17.3, 17.4, 17.7 | 57 | 15.2, 15.3 |
| 18 | 30.1, 30.3, 30.4 | 38 | 24.1, 24.2 | 58 | 28.1, 28.3, 28.4, 28.5 |
| 19 | 21.1, 22.1 | 39 | 9.3, 12.1, 12.2 | 59 | Section 10, 16.4 |
| 20 | 18.1, 18.2, 18.3, 19.1 | 40 | 19.3 | 60 | 19.2 |

Standards Correlation Chart (Teacher's Edition)

The chart below correlates each standard for the Algebra I course as given in the 2016 Mississippi College- and Career-Readiness Standards for Mathematics. The Text Section column gives the Section in the text where each standard is reviewed (including scaffolding skills). The Practice Test columns give the question number(s) in each Practice Test that correlate to each standard.

| MS CCRS Algebra I Standards | Text Section(s) | Practice Test 1 | Practice Test 2 |
|---|--|-----------------|-----------------|
| <p>Number and Quantity</p> <p>The Real Number System (N-RN) Use properties of rational and irrational numbers</p> <p>N-RN.3 Explain why:</p> <ul style="list-style-type: none"> • the sum or product of two rational numbers is rational; • the sum of a rational number and an irrational number is irrational; and • the product of a nonzero rational number and an irrational number is irrational. | Section 1 | 1 | 3 |
| <p>Quantities (N-Q) Reason quantitatively and use units to solve problems</p> <p>N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.</p> | Sections 3, 11, 20, 24, 25, 26, 29, 31 | 3 | 1 |
| <p>N-Q.2 Define appropriate quantities for the purpose of descriptive modeling.</p> | Sections 3, 5, 11, 12, 13, 24, 28, 30, 31 | | |
| <p>N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.</p> | Sections 5, 11, 12, 13, 14, 15, 20, 24, 25, 29, 30, 31 | 9 | 2 |
| <p>Algebra</p> <p>Seeing Structure in Expressions (A-SSE) Interpret the structure of expressions</p> <p>A-SSE.1 Interpret expressions that represent a quantity in terms of its context.</p> <ol style="list-style-type: none"> a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.</i> | Sections 1, 2, 5, 17, 18, 24 | 22 | 21 |
| <p>A-SSE.2 Use the structure of an expression to identify ways to rewrite it. <i>For example, see $x^2 - y^2$ as $(x)^2 - (y)^2$ thus recognizing it as a difference of squares that can be factored as $(x - y)(x + y)$.</i></p> | Sections 2, 18 | 47 | 33 |
| <p>Write expressions in equivalent forms to solve problems</p> <p>A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.</p> <ol style="list-style-type: none"> a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. <i>For example the expression 1.15^t can be rewritten as $[1.15^{1/12}]^{12t} \sim 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</i> | Sections 2, 18, 19, 21, 22 | 11, 12, 50 | 7, 19, 24, 32 |

Standards Correlation Chart, continued

| MS CCRS Algebra I Standards | Text Section(s) | Practice Test 1 | Practice Test 2 |
|---|---|-----------------|-----------------|
| <p>Arithmetic with Polynomials and Rational Expressions (A-APR) Perform arithmetic operations on polynomials</p> <p>A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.</p> | Sections 2, 17 | 4, 10, 37 | 13, 17 |
| <p>Understand the relationship between zeros and factors of polynomials</p> <p>A-APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial (limit to 1st- and 2nd- degree polynomials).</p> | Sections 8, 19, 21, 23 | 5, 7, 20 | 6, 8, 46 |
| <p>Creating Equations (A-CED) Create equations that describe numbers or relationships</p> <p>A-CED.1 Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i></p> | Sections 3, 5, 19, 20 | 23 | |
| <p>A-CED.2 Create equations in two variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. <i>[Note this standard appears in future courses with a slight variation in the standard language.]</i></p> | Sections 3, 8, 11, 12, 14, 15, 21, 22, 23, 24 | 24 | 22 |
| <p>A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i></p> | Sections 5, 12, 13, 20, 24 | 15 | 30 |
| <p>A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. <i>For example, rearrange Ohm's law $V = IR$ to highlight resistance R.</i></p> | Sections 5, 8 | 21, 33 | 12, 36 |
| <p>Reasoning with Equations and Inequalities (A-REI) Understand solving equations as a process of reasoning and explain the reasoning</p> <p>A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.</p> | Section 4 | 34 | 39 |
| <p>Solve equations and inequalities in one variable</p> <p>A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.</p> | Sections 3, 4, 5 | 14, 46 | 4, 37 |
| <p>A-REI.4 Solve quadratic equations in one variable.</p> <ol style="list-style-type: none"> Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions. | Sections 19, 20 | 40, 60 | 9, 41 |

Standards Correlation Chart, continued

| MS CCRS Algebra I Standards | Text Section(s) | Practice Test 1 | Practice Test 2 |
|--|---------------------------------|-----------------|-----------------|
| <p>Solve systems of equations</p> <p>A-REI.5 Given a system of two equations in two variables, show and explain why the sum of equivalent forms of the equations produces the same solution as the original system.</p> | Section 13 | 31 | 15 |
| <p>A-REI.6 Solve systems of linear equations algebraically, exactly, and graphically while focusing on pairs of linear equations in two variables.</p> | Sections 8, 9, 13 | 32 | 34 |
| <p>Represent and solve equations and inequalities graphically</p> <p>A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).</p> | Sections 6, 7, 8, 9, 13, 14, 25 | 28, 35 | 5, 40 |
| <p>A-REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, quadratic, absolute value, and exponential functions.</p> | Sections 13, 27 | 45, 48 | 42, 49 |
| <p>A-REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</p> | Sections 12, 13 | 17, 39, 44 | 43, 47, 59 |
| <p>Functions</p> <p>Interpreting Functions (F-IF) Understand the concept of a function and use function notation</p> <p>F-IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.</p> | Sections 3, 6, 7 | | 48 |
| <p>F-IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</p> | Sections 6, 7, 11, 14, 15 | 36, 55 | 45 |
| <p>F-IF.3 Recognize that sequences are functions whose domain is a subset of the integers.</p> | Sections 10, 16 | 59 | 60 |
| <p>Interpret functions that arise in applications in terms of the context</p> <p>F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i></p> | Sections 21, 22, 23, 24, 25 | 41, 51 | 25, 26, 52 |
| <p>F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.</i></p> | Sections 1, 6, 7, 9, 11, 14, 24 | 43 | 56 |
| <p>F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.</p> | Sections 11, 23 | 56 | 53 |

Standards Correlation Chart, continued

| MS CCRS Algebra I Standards | Text Section(s) | Practice Test 1 | Practice Test 2 |
|---|---|-----------------|-----------------|
| <p>Analyze functions using different representations</p> <p>F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</p> <ol style="list-style-type: none"> Graph functions (linear and quadratic) and show intercepts, maxima, and minima. Graph square root and piecewise-defined functions, including absolute value functions. | Sections 6, 7, 8, 9, 11, 14, 21, 22, 23, 25 | 19, 54 | 54, 58 |
| <p>F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</p> <ol style="list-style-type: none"> Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. | Sections 7, 8, 9, 19, 22, 24 | 38 | 28 |
| <p>F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i></p> | Sections 18, 23, 24 | 52 | 10 |
| <p>Building Functions (F-BF) Build a function that models a relationship between two quantities</p> <p>F-BF.1 Write a function that describes a relationship between two quantities.</p> <ol style="list-style-type: none"> Determine an explicit expression or steps for calculation from a context. | Sections 10, 14, 15, 16 | 26 | 29 |
| <p>Build new functions from existing functions</p> <p>F-BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. <i>Include recognizing even and odd functions from their graphs and algebraic expressions for them.</i></p> | Sections 6, 26, 28 | 30, 42, 58 | 44, 50, 51 |
| <p>Linear, Quadratic, and Exponential Models (F-LE) Construct and compare linear, quadratic, and exponential models and solve problems</p> <p>F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.</p> <ol style="list-style-type: none"> Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. | Sections 8, 9, 10, 11, 14, 15, 16 | 13, 25 | 27, 57 |
| <p>F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table)</p> | Sections 8, 9, 10, 11, 14, 15, 16 | 16, 27, 53 | 11, 14, 55 |
| <p>Interpret expressions for functions in terms of the situation they model</p> <p>F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context.</p> | Sections 8, 9, 11, 14, 15, 16 | 57 | 38 |

Standards Correlation Chart, continued

| MS CCRS Algebra I Standards | Text Section(s) | Practice Test 1 | Practice Test 2 |
|--|-----------------------|-----------------|-----------------|
| <p>Statistics and Probability</p> <p>Interpreting Categorical and Quantitative Data (S-ID) Summarize, represent, and interpret data on a single count or measurement variable</p> <p>S-ID.1 Represent and analyze data with plots on the real number line (dot plots, histograms, and box plots).</p> | Sections 30, 31 | 8 | 18 |
| <p>S-ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.</p> | Sections 30, 31 | 18 | |
| <p>S-ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).</p> | Sections 30, 31 | | 31, 35 |
| <p>Summarize, represent, and interpret data on two categorical and quantitative variables</p> <p>S-ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.</p> | Section 32 | 49 | 20 |
| <p>S-ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.</p> <p>a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. <i>Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.</i></p> <p>b. Informally assess the fit of a function by plotting and analyzing residuals.</p> <p>c. Fit a linear function for a scatter plot that suggests a linear association.</p> | Section 29 | 6 | 23 |
| <p>Interpret linear models</p> <p>S-ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.</p> | Sections 8, 9, 11, 29 | 29 | 16 |
| <p>S-ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.</p> | Section 29 | | |
| <p>S-ID.9 Distinguish between correlation and causation.</p> | Section 29 | | |